Testing the Standard Model with lattice QCD and effective field theory

Jon A. Bailey

SWME Collaboration

Yong-Chull Jang, Hyung-Jin Kim, Jongjeong Kim, Weonjong Lee, Jaehoon Leem, Sungwoo Park, Stephen R. Sharpe, Boram Yoon

Fermilab Lattice and MILC Collaborations

Daping Du, A. X. El-Khadra, Steven Gottlieb, R. D. Jain, A. S. Kronfeld, J. Laiho, Yuzhi Liu, E. T. Neil, J. Simone, R. S. Van de Water, R. Zhou

October 30, 2014

- $m_{\pi/K}$ and $f_{\pi/K}$ in mixed-action χPT (Hyung-Jin Kim, Jongjeong Kim, Boram Yoon)
- BSM contributions to $K - \overline{K}$ mixing (Jaehoon Leem)
- $|V_{cb}|$ from $B \to D^* \ell \nu$ at zero recoil (Yong-Chull Jang, Jaehoon Leem)

Testing the Standard Model with lattice QCD and effective field theory

Jon A. Bailey

SWME Collaboration

Yong-Chull Jang, Hyung-Jin Kim, Jongjeong Kim, Weonjong Lee, Jaehoon Leem, Sungwoo Park, Stephen R. Sharpe, Boram Yoon

Fermilab Lattice and MILC Collaborations

Daping Du, A. X. El-Khadra, Steven Gottlieb, R. D. Jain, A. S. Kronfeld, J. Laiho, Yuzhi Liu, E. T. Neil, J. Simone, R. S. Van de Water, R. Zhou

October 30, 2014

- $|V_{ub}|$ from $B \to \pi \ell \nu$, rare decay $B \to \pi \ell^+ \ell^-$ (Daping Du)
- $|V_{ub}|$ from $B_s \to K \ell \nu$ (Yuzhi Liu)
- $|V_{cs}|$ from $D \to K \ell \nu$, $|V_{cd}|$ from $D \to \pi \ell \nu$ (JAB)
- Rare decay $B \to K \ell^+ \ell^-$ (R. Zhou)

Testing the Standard Model with lattice QCD and effective field theory

Jon A. Bailey

SWME Collaboration

Yong-Chull Jang, Hyung-Jin Kim, Jongjeong Kim, Weonjong Lee, Jaehoon Leem, Sungwoo Park, Stephen R. Sharpe, Boram Yoon

Fermilab Lattice and MILC Collaborations

Daping Du, A. X. El-Khadra, Steven Gottlieb, R. D. Jain, A. S. Kronfeld, J. Laiho, Yuzhi Liu, E. T. Neil, J. Simone, R. S. Van de Water, R. Zhou

October 30, 2014

Goals

- Quark flavor physics entering exciting era
  - Precision experiments and theoretical calculations → stringent tests of SM, searches for new physics
  - Many opportunities
- Understand physics of quark flavor, CP violation
  - Test SM picture, CKM unitarity
  - Physics responsible for CKM matrix elements
- Search for new physics
  - Rare decays
  - Differences ≥ 3σ :
    - $|V_{ub}|_{excl.}$ and $|V_{ub}|_{incl.}$
    - $|V_{cb}|_{excl.}$ and $|V_{cb}|_{incl.}$
    - $|\varepsilon_K|$ from SM and experiment [SWME, Lattice 2014]
    - BaBar excess in $R(D^{(*)}) = BR(B \to D^{(*)}\tau\nu) / BR(B \to D^{(*)}l\nu)$ [FNAL/MILC, PRL 2012, arXiv:1206.4992; Lees *et al*., PRL 2012, arXiv:1205.5442]
- Constrain, characterize new physics

Lattice systematics

- Discretization effects
  - Light quarks, gluons
  - Heavy quarks
- Unphysically large light (u, d) quark masses
- Operator matching
- Finite-volume effects
- Scale fixing
- Quark mass tuning
- Fitting
- Electroweak corrections ~ *e.g.*, EM isospin-breaking
- Quenching ~ omitting s, c vacuum polarization

Lattice systematics

- Discretization effects
  - Light quarks, gluons
  - Heavy quarks
- Unphysically large light (*u*, *d*) quark masses
- Operator matching
- Finite-volume effects
- Scale fixing
- Quark mass tuning
- Fitting
- Electroweak corrections ~ *e.g.*, EM isospin-breaking
- ~~Quenching ~ omitting *s*, *c* vacuum polarization~~

Effective field theories

Lattice systematics

- Discretization effects
  - Light quarks, gluons
  - Heavy quarks
- Unphysically large light (*u*, *d*) quark masses
- Operator matching
- Finite-volume effects
- Scale fixing
- Quark mass tuning
- Fitting
- Electroweak corrections ~ *e.g.*, EM isospin-breaking
- ~~Quenching ~ omitting *s*, *c* vacuum polarization~~

Effective field theories

Effective field theories

- Effective continuum field theories
  - Match to lattice gauge theory
  - Accelerate approach to continuum limit (improvement programs)
  - Propagate systematics to observables
  - Symanzik effective theory, heavy quark effective theory
- Lattice (staggered, …) chiral perturbation theory
  - Match to lattice gauge theory, scales $E << \Lambda_{\chi SB}$
  - Hadronic degrees of freedom, interactions
  - Extrapolate light (u, d) quark mass, lattice spacing

Symanzik effective theory

[Symanzik, NPB 226 (1983) 187; *ibid.*, 205]

- Continuum spacetime and quark, gluon degrees of freedom
- Light quarks ~ lattice spacing (am_q << 1)
- Lattice symmetries ~ hypercubic spacetime rotations
- Expand action, operators ~ engineering dimension

$$S_{\mathrm{LGT}} \doteq S_{\mathrm{Sym}} = S_{\mathrm{QCD}} + aS_5 + a^2 S_6 + a^3 S_7 + \ldots$$

$$E \ll a^{-1}$$

- Match on-shell correlation functions to lattice gauge theory
- Discretization effects perturbations of continuum limit theory
- Assumption of light quarks can be lifted (arbitrary am_q) [Aoki et al., Prog. Theor. Phys. 2003, hep-lat/0107009; Christ et al., PRD 2007, hep-lat/0608006]

Heavy quark effective theory

[Kronfeld, PRD 2000, hep-lat/0002008; Harada *et al.*, PRD 2002, 2005, hep-lat/0112044, hep-lat/0112045]

- Lattice gauge theory with heavy quark symmetry
  - $m_Q >> \Lambda_{\mathrm{QCD}}$
  - Arbitrary am_Q
- Continuum spacetime, heavy quark, gluon degrees of freedom
- Lattice symmetries ~ hypercubic spacetime rotations
- Expand action, operators ~ heavy quark power counting

$$a\Lambda_{\mathrm{QCD}} \sim \frac{\Lambda_{\mathrm{QCD}}}{m_Q} \ll 1$$

$$S_{\mathrm{LGT}} \doteq S_{\mathrm{HQET}} = S^{(0)} + S^{(1)} + S^{(2)} + \dots$$

- Match on-shell correlation functions to LGT and QCD
- Discretization effects perturbations of continuum limit theory

Staggered χPT

[Lee and Sharpe, PRD 1999, hep-lat/9905023; Aubin and Bernard, PRD 2003, hep-lat/0304014, hep-lat/0306026]

- Consider Symanzik effective continuum theory for $E << \Lambda_{\chi\mathrm{SB}}$
- Spontaneously broken chiral SU(2) or SU(3)
- Map operators of SET to χPT ~ lattice χPT
  - Remnant doubler degrees of freedom ~ pseudo-flavor = taste
  - Lattice symmetries ~ hypercubic rotations, taste symmetry
- Expand action, operators about chiral, continuum limits

$$a\Lambda_{\mathrm{QCD}} \sim \Lambda_{\mathrm{QCD}}/\Lambda_{\chi\mathrm{SB}} \ll 1$$
$$S_{\mathrm{LGT}} \doteq S_{\mathrm{Sym}} \doteq S_{\mathrm{L}\chi\mathrm{PT}} = S_{\mathrm{LO}} + S_{\mathrm{NLO}} + \ldots$$

- Discretization effects of light quarks, gluons and deviations from chiral symmetry as perturbations
- Fit numerical simulation data to extract LECs, extrapolate
- Variations for different lattice fermions, heavy (*c*, *b*) quarks, ...

$m_{\pi/K}$ and $f_{\pi/K}$ in mixed-action SχPT

[JAB *et al*., Lattice 2013, arXiv:1311.6268]

- f_K / f_π and $K \to \pi l \nu$ form factor: $|V_{us}|$; Gasser-Leutwyler couplings, LGT LECs; $m_{\pi/K}$ and $f_{\pi/K}$ ~ light quark masses, scale
- Mixed-action lattice QCD ~ Symanzik imp. action to reduce valence quark cutoff effects, low cost vacuum polarization
- Mixed-action lattice (staggered …) chiral perturbation theory
  - Symanzik effective theory for mixed-action lattice theory
  - Valence-sea symmetry broken
  - Map to operators of chiral effective theory, calculate $m_{\pi/K}$ and $f_{\pi/K}$
- Calculated one-loop (NLO) corrections for all lattice irreps (meson tastes), valence-valence, valence-sea mesons
  - Taste-pseudoscalar valence-valence mesons ~ Goldstone bosons
  - Results for valence-valence $m_{\pi/K}$, all $f_{\pi/K}$ ~ form of unmixed theory

B parameters for kaon mixing BSM

- Constrain new physics entering neutral kaon mixing
- *B* parameters enter matrix elements of ΔS = 2 effective Hamiltonian

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \sum_{i=1}^{5} C_i(\mu) Q_i(\mu)$$

$$B_i(\mu) = \frac{\langle \overline{K}_0 | Q_i(\mu) | K_0 \rangle}{N_i \langle \overline{K}_0 | \bar{s}\gamma_5 d(\mu) | 0 \rangle \langle 0 | \bar{s}\gamma_5 d(\mu) | K_0 \rangle}$$

$$Q_1 = [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1-\gamma_5) d^b]$$

$$(N_2,\ N_3,\ N_4,\ N_5) = (5/3,\ 4,\ -2,\ 4/3)$$

$$Q_2 = [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1-\gamma_5) d^b]$$

$$Q_3 = [\bar{s}^a \sigma_{\mu\nu} (1-\gamma_5) d^a][\bar{s}^b \sigma_{\mu\nu} (1-\gamma_5) d^b]$$

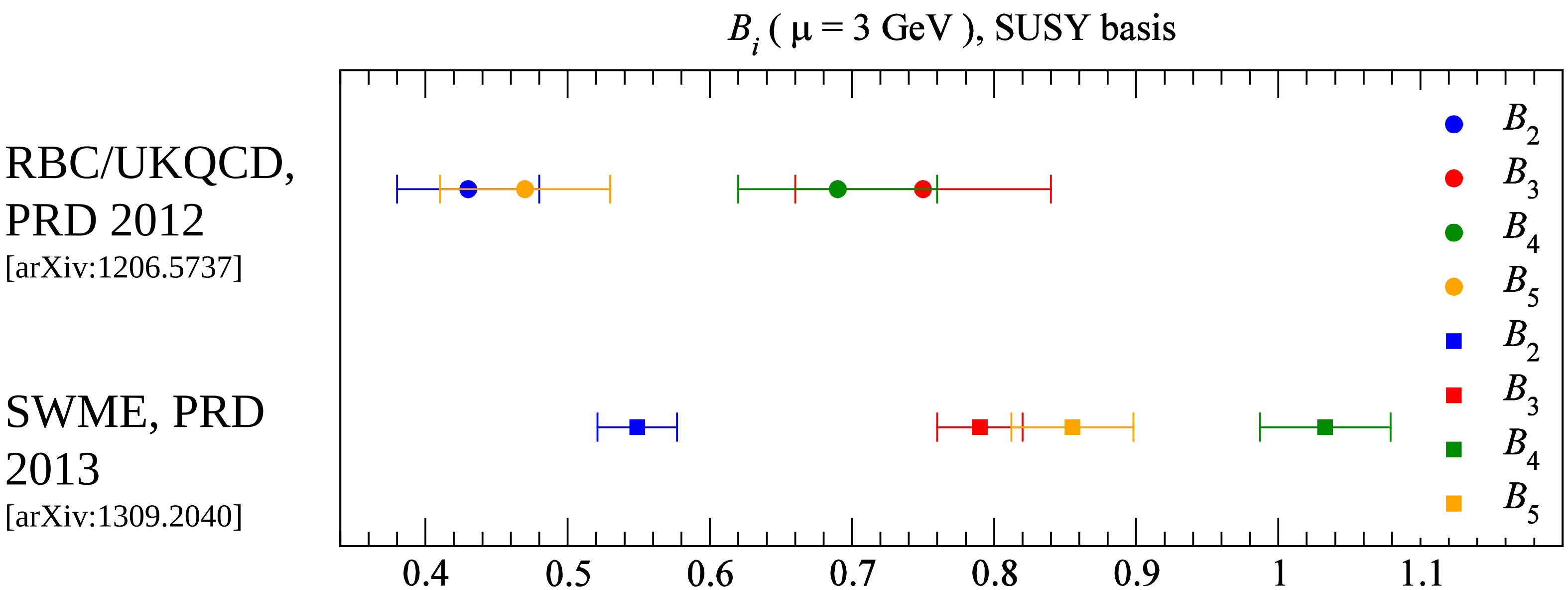
$$Q_4 = [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1+\gamma_5) d^b]$$

$$Q_5 = [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1+\gamma_5) d^b]$$

- Gauge ensembles with 2+1 flavors of asqtad-improved staggered quarks generated by MILC Collaboration
- HYP-improved staggered valence quarks
- SWME, PRD 2013: First calculation with multiple lattice spacings (~ 0.09, 0.06, 0.045 fm) and vacuum polarization of *u*, *d*, *s* quarks [Bae *et al.*, arXiv:1309.2040]

B parameters for kaon mixing BSM

- Light (*u*, *d*) sea quark mass 0.4 to 0.1m_s, valence quark masses 1 to 0.1m_s
- Identify golden ratios with mixed-action SχPT [JAB *et al.*, PRD 2012, arXiv:1202.1570]
  - Eliminate NLO chiral logarithms, with discretization effects
  - Simplify extrapolation to physical light mass, continuum limit



- Results for B_K , B_3 agree with those of 2+1 flavor domain-wall calculation at lattice spacing ~ 0.086 fm
- Results for B_2, B_4, B_5 disagree (also ETMC); investigating differences

B parameters for kaon mixing BSM

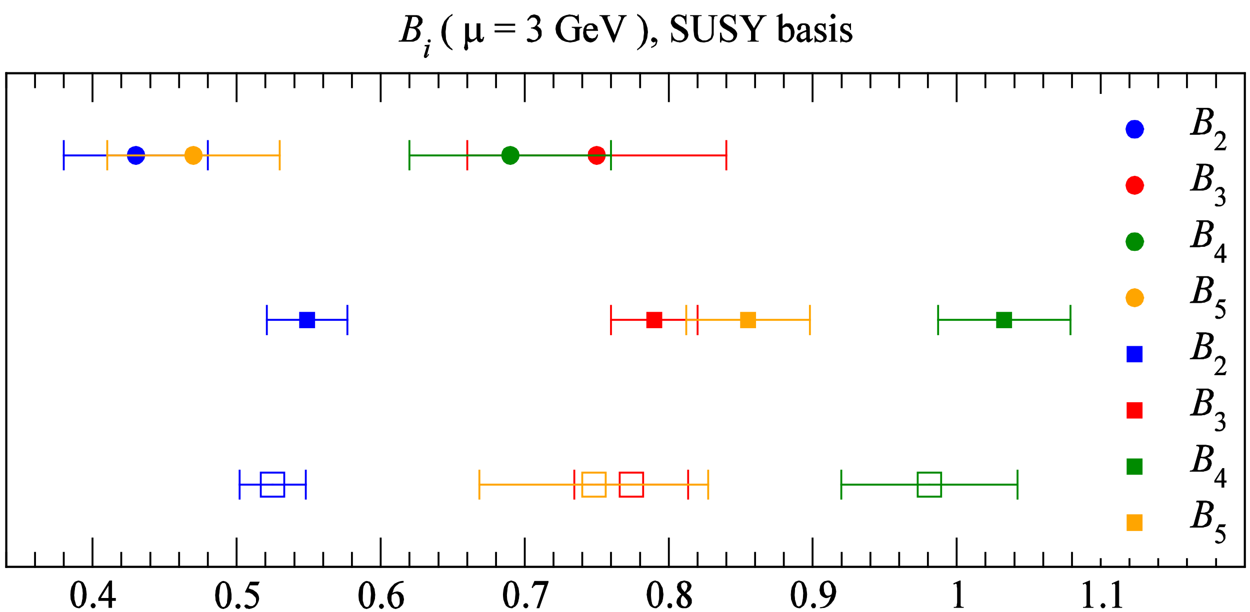
[Jaehoon Leem *et al.*, Lattice 2014]

- Update with additional ensembles: Light (u, d) sea quark mass 0.6 to 0.05m_s
- Lattice spacings ~ (0.12), 0.09, 0.06, 0.045 fm

RBC/UKQCD, PRD 2012

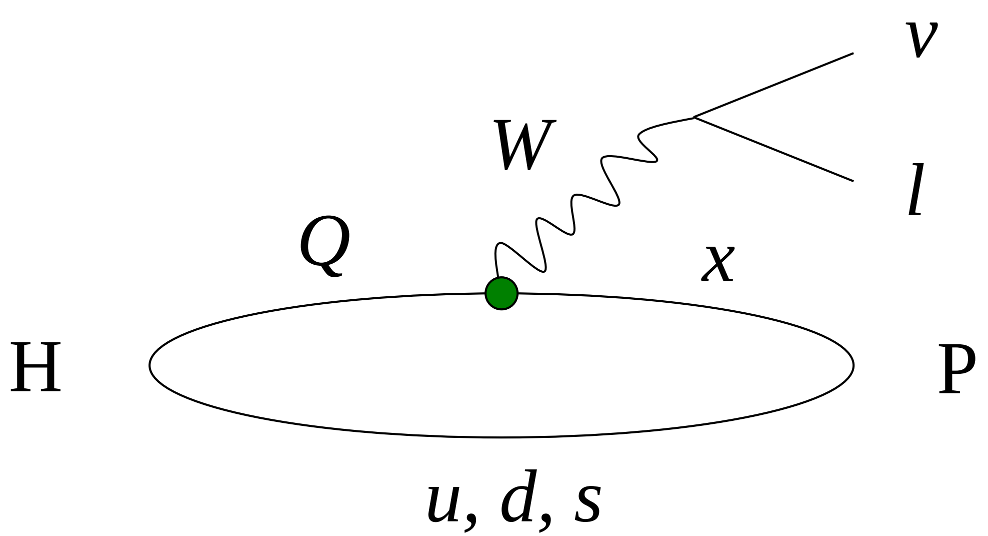
SWME, PRD 2013

SWME, update 2014, prelim., [JHL, KPS 2014]



- Systematics from chiral-continuum extrapolation, perturbative renormalization
- Differences in B_2, B_4, B_5 persist
- Overall picture remains same

CKM matrix elements from H → P$l\nu$

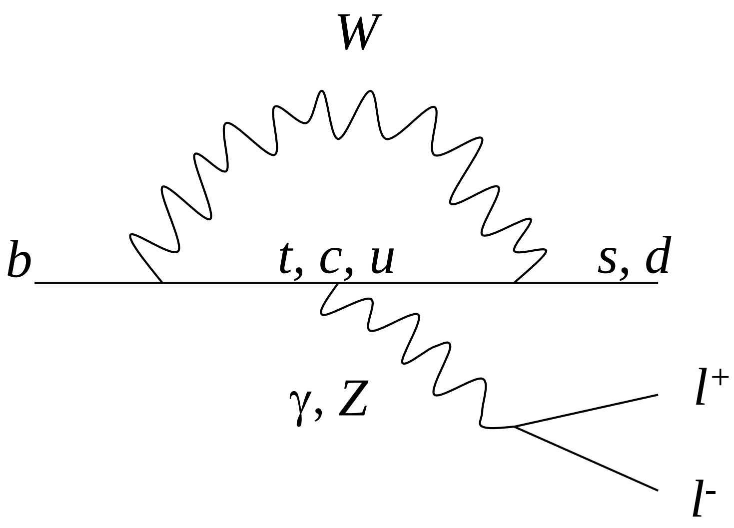


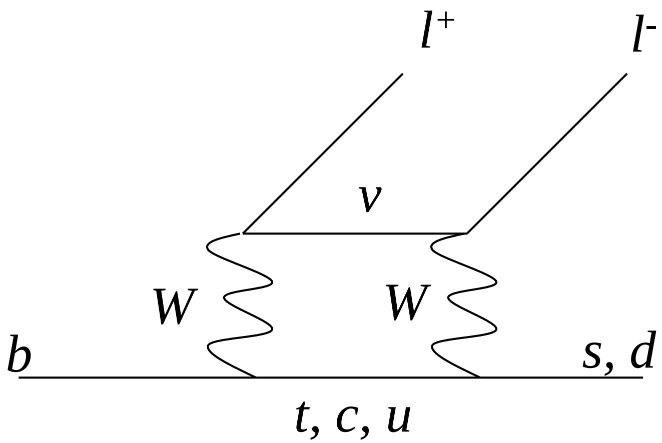
$H \to P$	$Q \to x$	CKM
$B \to \pi$	$b \to u$	$\|V_{ub}\|$
$B_s \to K$	$b \to u$	$\|V_{ub}\|$
$D \to \pi$	$c \to d$	$\|V_{cd}\|$
$D \to K$	$c \to s$	$\|V_{cs}\|$

- Heavy quark Q decays into light quark x
- Mediated by vector current in SM
- Scalar and tensor currents enter BSM

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}(\mathrm{CKM})^2|\mathbf{p}_P|^3|f_+(q^2)|^2$$

Rare decays B → Pl^+l^-





- Loop suppressed in SM ~ FCNC, prime candidates for new physics
- Vector, scalar, and tensor currents enter BSM *via* effective Lagrangian

$H \to Pl\nu$, $H \to Pl^+l^-$ form factors

- Defined in terms of hadronic matrix elements of flavor-changing vector, scalar, tensor currents

$$\langle P(k)|\bar{x}\gamma^\mu Q|H(p)\rangle = f_+(q^2)\left[p^\mu + k^\mu - \frac{M_H^2 - M_P^2}{q^2}q^\mu\right] + f_0(q^2)\frac{M_H^2 - M_P^2}{q^2}q^\mu$$

$$= \sqrt{2M_H}[v^\mu f_\parallel(E_P) + k_\perp^\mu f_\perp(E_P)]$$

$$\langle P(k)|\bar{x}Q|H(p)\rangle = f_0(q^2)\frac{M_H^2 - M_P^2}{m_Q - m_x}$$

$$\langle P(k)|\bar{x}i\sigma^{\mu\nu}Q|H(p)\rangle = 2f_T(q^2)\frac{p^\mu k^\nu - p^\nu k^\mu}{M_H + M_P}$$

$$q^2 \equiv (p-k)^2 = M_H^2 + M_P^2 - 2M_H E_P$$

$$v^\mu \equiv p^\mu / M_H$$

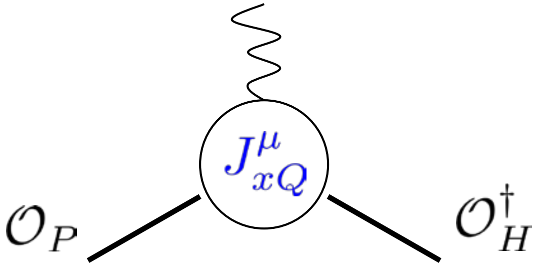
$$k_\perp^\mu \equiv k^\mu - (v \cdot k)v^\mu$$

- q is momentum transferred to outgoing leptons
- E_P is energy of recoiling P-meson, in H rest frame
- $f_\parallel$, $f_\perp$ parameterization convenient in HQET, ChPT, lattice QCD
- f_T related to $f_\perp$ by HQS

Reduction procedure ~ form factors

- Hadronic matrix elements simply related to 3-point Green functions ~ vacuum expectation values of currents between creation, annihilation operators for initial, final mesons
- Meson propagators, 2-point Green functions, provide amputation factors, kinematic factors
- Euclidean Green functions of QCD ~ correlation functions of lattice gauge theory (Monte Carlo estimators)

$$\langle P(k)|\bar{x}\gamma^\mu Q|H(p)\rangle \sim \langle 0|T\mathcal{O}_P\,\bar{x}\gamma^\mu Q\,\mathcal{O}_H^\dagger|0\rangle$$



- Correlation function behavior well understood ~ series of exponentials
- Fit 3-point and 2-point correlation functions to extract amputation, kinematic factors and desired hadronic matrix elements
  - Appropriately constructed ratios of correlation functions ~ form factors; simple fits to constant, constant + leading excited-state contribution
  - Simultaneous fits to 3-point and 2-point correlators ~ greater control over excited-state contributions

Form factor lattice calculations

- Generate correlation functions (lattice data) at different lattice spacings, quark masses, recoil momenta
- Fit correlation functions to obtain form factors at different lattice spacings, quark masses, recoil momenta
- Renormalize currents (match to continuum normalization)
  - Not needed (automatic) if CVC/PCAC relation holds in lattice theory
  - Necessary for Fermilab bottom, charm
- Fit form factors as functions of lattice spacing, quark masses, recoil momenta and extrapolate to continuum limit, physical quark masses
  - Model-independent parameterization desirable
  - Staggered chiral perturbation theory, in chiral regime
- Incorporate systematic errors
- Interpolate and (for B decays) extrapolate recoil energy dependence
  - Model-independent parameterization
  - z-expansion derived from analyticity, unitarity, crossing symmetry, heavy quark symmetry

Simulation details

- Quark and gluon actions
  - Gluon action: one-loop Symanzik-improved Luscher-Weisz gauge action [Weisz, NPB 1983; Curci et al., PLB 1983; Weisz and Wohlert, NPB 1984; Luscher and Weisz, PLB and CMP 1985]
  - Fermion actions
    - *u*, *d*, *s* quarks: $O(a^2)$ tadpole improved (asqtad) staggered action [Alford et al., PLB 1995; Bernard et al., PRD 1998; MILC, PRD 1999; Lepage, PRD 1999]
    - *c*, *b* quarks: Sheikholeslami-Wohlert (clover) action with Fermilab interpretation [El-Khadra et al., PRD 1997; Kronfeld, PRD 2000]
  - Gauge and staggered actions used (MILC) to generate 2+1 flavor asqtad staggered gauge ensembles, with fourth root of staggered determinant [Bazavov et al., RMP 2010]
  - Fermilab method uses heavy quark symmetry, controls discretization effects of charm and bottom quarks ~ validate B form factor lattice calculations with SM values of CKM matrix elements and D semileptonic branching fractions
- Input parameters
  - *u*, *d*, *s* quark masses from π, K masses (isospin limit)
  - *c*, *b* quark masses from D_s, B_s masses (spin-averaged kinetic masses)
  - Scale determination from f_π *via* modified Sommer scale (r_1)
- Valence masses, ensemble parameters
  - *s* quark mass approximately physical on different ensembles
  - *u*, *d* quark masses vary for different projects, typical range ~ 0.4 to 0.1 or $0.05 m_s$
  - Lattice spacings typical range ~ 0.12 to 0.06 or 0.045 fm
- Blind analyses of correlator data, form factors by introducing offset into current renormalization factors

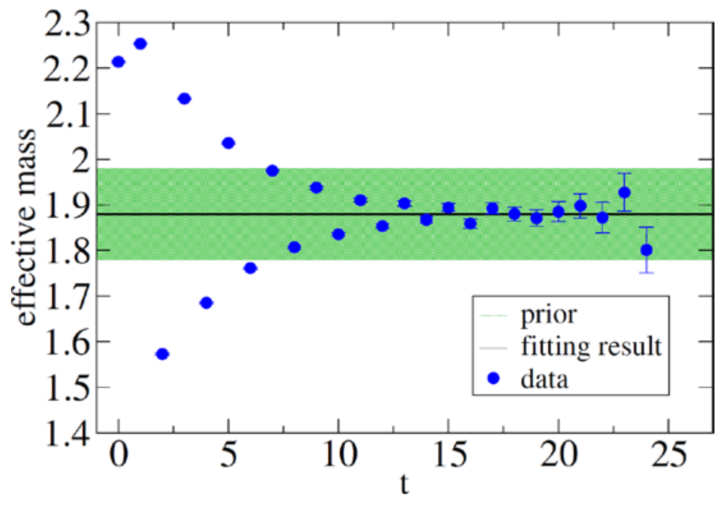
$|V_{ub}|$ from $B_s \to Kl\nu$

[Yuzhi Liu, R. Zhou *et al.*, Lattice 2013, arXiv:1312.3197]

- Lattice QCD form factors + measurements by Belle II, LHCb → $|V_{ub}|$
- Study of excited-state contamination in ratios
- Simultaneous fits of 2-point, 3-point correlators to three exponentials (with staggered oscillating partners and finite-volume effects)
- Select fit intervals, priors from effective mass, stability plots
- Hard kaon, SU(2) staggered ChPT fits in progress

$$C_{ss'}(t) = \sum_{n=0}^{N-1} \Big[A_{ns} A_{ns'} \left(e^{-E_n t} + e^{-E_n (N_T - t)} \right) - (-1)^t A'_{ns} A'_{ns'} \left(e^{-E'_n t} + e^{-E'_n (N_T - t)} \right) \Big]$$

$$C^{\mu}_{ss'}(t, T) = \sum_{m,n=0}^{N-1} (-1)^{mt} (-1)^{n(T-t)} \times A^K_{ms} V^{\mu}_{mn} A^{B_s}_{ns'} e^{-E^m_K t} e^{-M^n_{B_s}(T-t)}$$



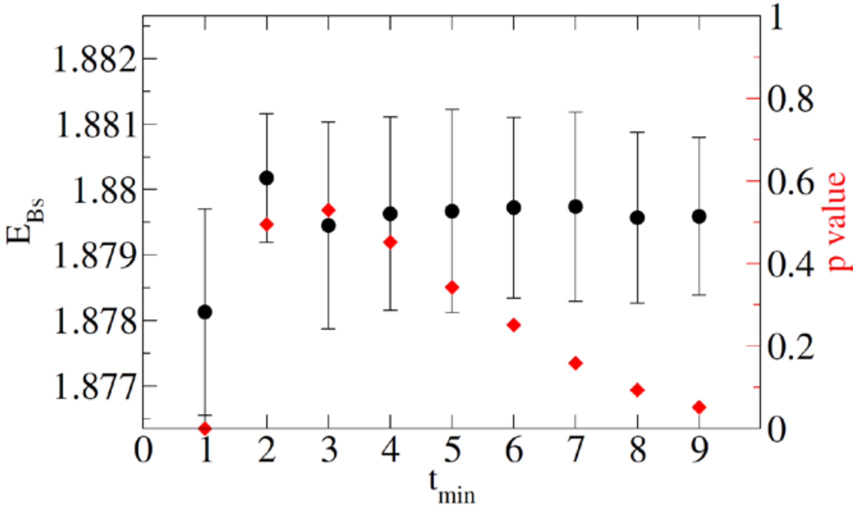
$|V_{ub}|$ from $B_s \to Kl\nu$

[Yuzhi Liu, R. Zhou *et al.*, Lattice 2013, arXiv:1312.3197]

- Lattice QCD form factors + measurements by Belle II, LHCb → $|V_{ub}|$
- Study of excited-state contamination in ratios
- Simultaneous fits of 2-point, 3-point correlators to three exponentials (with staggered oscillating partners and finite-volume effects)
- Select fit intervals, priors from effective mass, stability plots
- Hard kaon, SU(2) staggered ChPT fits in progress

$$C_{ss'}(t) = \sum_{n=0}^{N-1} \Big[A_{ns} A_{ns'} \left(e^{-E_n t} + e^{-E_n (N_T - t)} \right) - (-1)^t A'_{ns} A'_{ns'} \left(e^{-E'_n t} + e^{-E'_n (N_T - t)} \right) \Big]$$

- Expect final errors ~ 5% level



$|V_{cs(d)}|$ from $D \to K(\pi)l\nu$

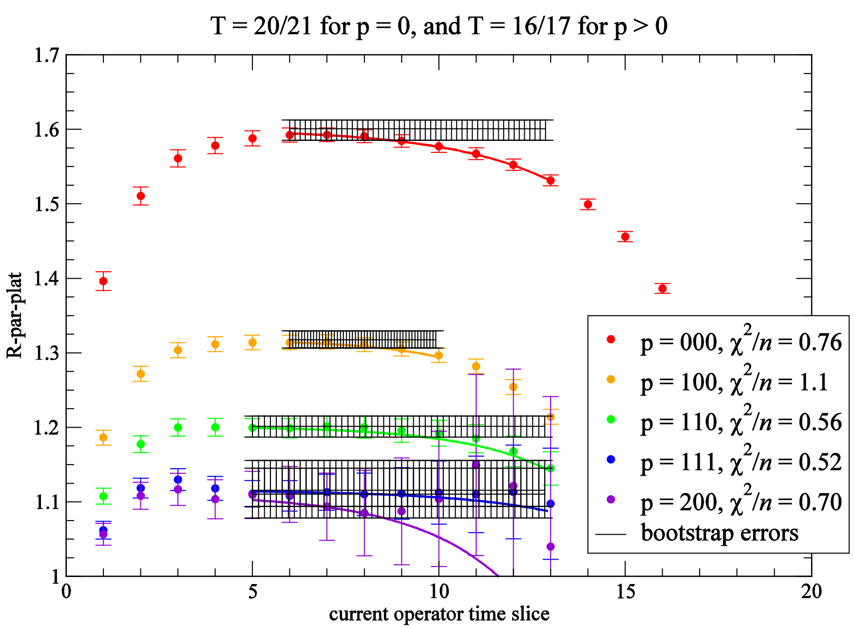
[JAB *et al.*, Lattice 2012, arXiv:1211.4964]

- To decrease statistical errors, ratios constructed with $\mathbf{p} = \mathbf{0}$ pion, kaon 2-points and continuum dispersion relations
- Take m_P, m_D from fits to 2-points
- Ratios asymptote to lattice form factors if continuum relations hold
  - Dispersion relations
  - Momentum independence of amputation factors

$$\frac{1}{\phi_\mu}\frac{\overline{C}_{3,\mu}^{D\to P}(t,T;\mathbf{p})}{\sqrt{\overline{C}_2^P(t;\mathbf{0})\overline{C}_2^D(T-t)}}\frac{\sqrt{m_P{}^2+\mathbf{p}^2}}{e^{-t\sqrt{m_P{}^2+\mathbf{p}^2}}}\sqrt{\frac{2e^{-m_P t}}{m_P e^{-m_D(T-t)}}}$$

$$\sim f^{\rm lat}_{\parallel,\perp}\left[\frac{Z_P(\mathbf{p})}{Z_P(\mathbf{0})}\frac{\sqrt{m_P^2+\mathbf{p}^2}}{E_P}e^{-(E_P-\sqrt{m_P^2+\mathbf{p}^2})t}\right]$$

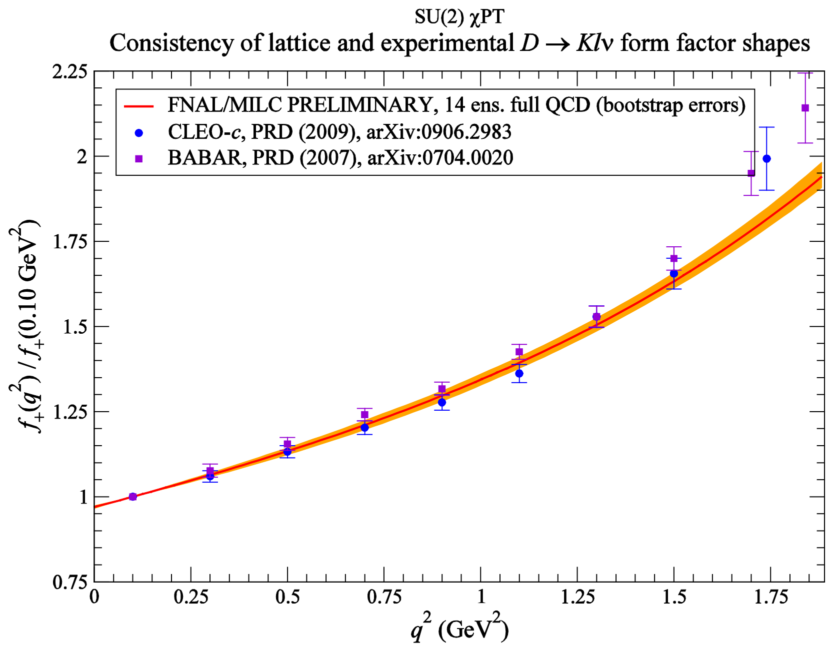
- Include leading excited-state contribution in fits to correlator ratios
- Selection of final ratio fit intervals, priors in progress



$|V_{cs(d)}|$ from $D \to K(\pi)l\nu$

[JAB *et al.*, Lattice 2012, arXiv:1211.4964]

- Preliminary fits to SU(3) and SU(2) staggered chiral perturbation theory, chiral-continuum extrapolation
- Cross checks of form factor shapes with CLEO, BaBar, Belle, …
  - Lattice QCD breaks down at large momenta; exp. limited at small momenta
  - For D semileptonic decays, kinematically allowed momenta accessible to both
  - Test lattice QCD against experimentally determined D form factor shapes
  - Validate application of lattice QCD methods to B semileptonic decays
- Blinding factors cancel in fiducial ratios, form factor normalizations remain hidden

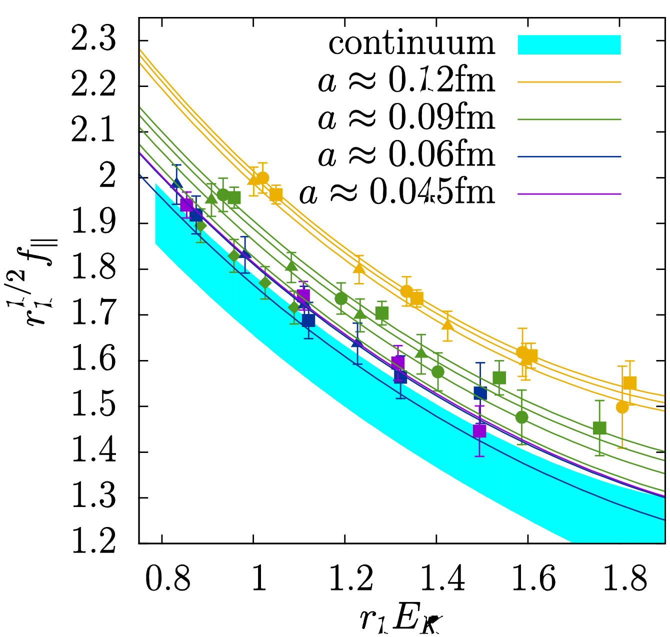


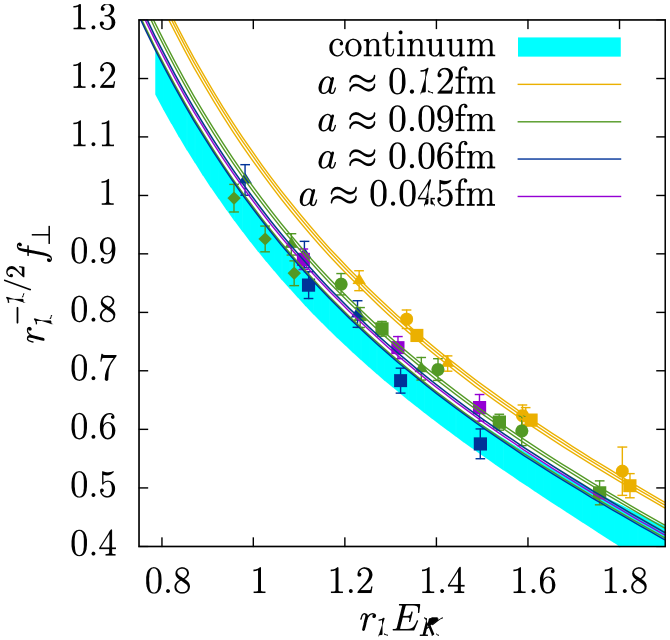
- Preliminary SU(2) chiral-continuum extrapolation
- Ratio of vector form factor for $D \to K$ decay to value at $q^2 = 0.10\ \mathrm{GeV}^2$ (arb.)
- Unity at fiducial point, by definition
- Statistical errors only in lattice result
- Reasonable qualitative agreement, except perhaps for high q^2 (low recoil)
- Quantitative comparison requires estimates of lattice systematics

Rare decay B → Kl^+l^-

[Yuzhi Liu, R. Zhou *et al.*, Lattice 2013, arXiv:1312.3197]

- New physics searches by Belle, BaBar, CDF, LHCb, Belle II
- Fit form factor ratios to constants ~ cross check for consistency with simultaneous fits
- SU(3) SChPT fails → SU(2) SChPT at NLO works well for chiral-continuum extrapolations; include analytic strange mass dependence
- Include uncertainty from B*Bπ coupling, heavy quark discretization effects, in ChPT

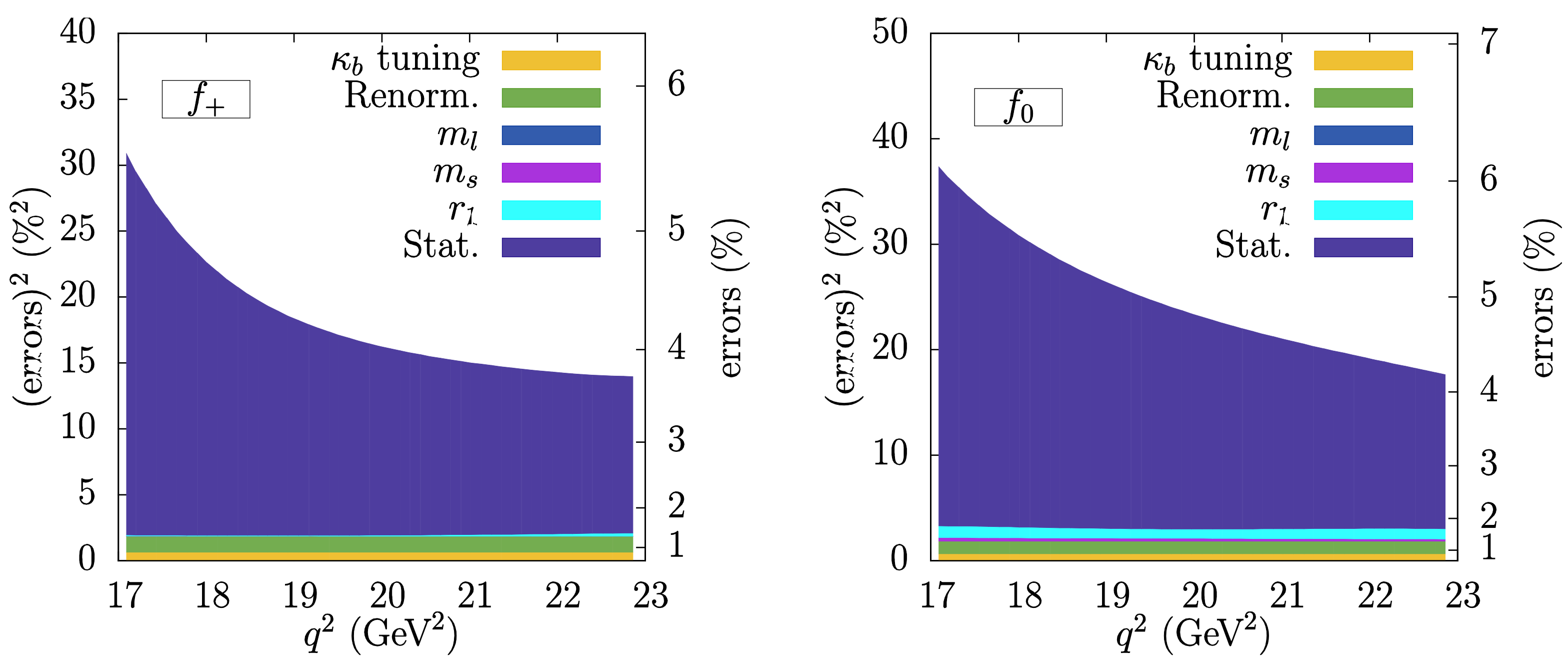




Rare decay $B \to K l^+ l^-$

[Yuzhi Liu, R. Zhou *et al.*, Lattice 2013, arXiv:1312.3197]

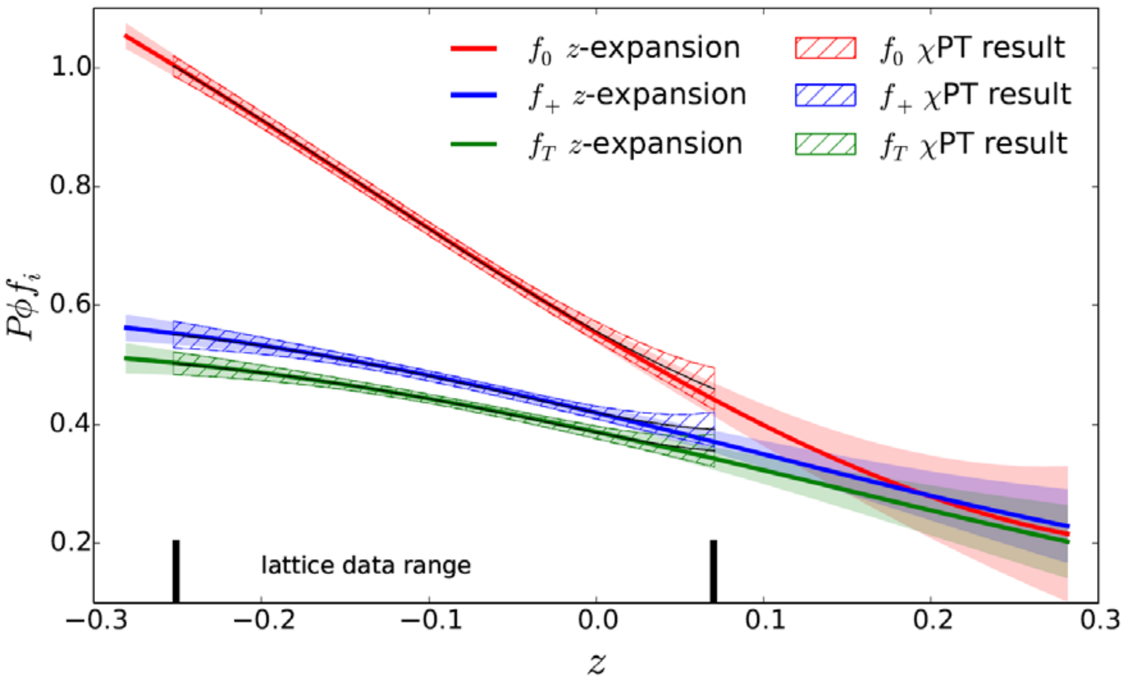
- Estimate systematics from fitting, current renormalization, scale determination, tuning input masses, finite-volume effects
- Model-independent *z*-expansion fit, extrapolation to large recoil energy
  - Kinematic constraint at $q^2 = 0$ imposed after cross check
  - Heavy-quark bound used to set priors on higher-order terms in *z*-expansion
  - 3-term expansion describes lattice data well, accounts for truncation
- Analysis complete; final errors 4-6% for $q^2 > 17$ GeV^2



B → π*lv*, B → π*l*⁺*l*⁻

[Daping Du *et al*., Lattice 2013, arXiv:1311.6552]

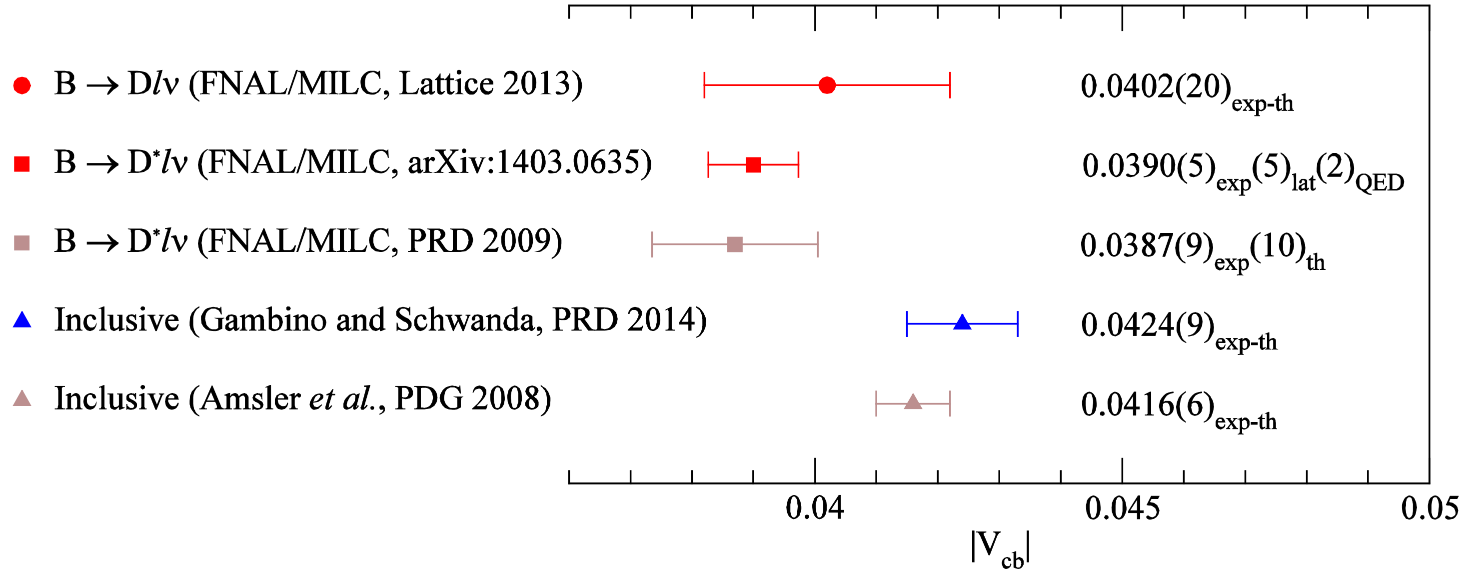
- Fit correlator ratios, including leading excited-state contribution
- Chiral-continuum extrapolation with hard pion SU(2) staggered ChPT
- Functional *z*-expansion to extrapolate data to larger recoil (4 terms)
- Systematic error estimates in progress for B → π*ll*
- B → π*lv* analysis complete, ~ 4% error for $|V_{ub}|$



$|V_{cb}|$ and quark flavor physics

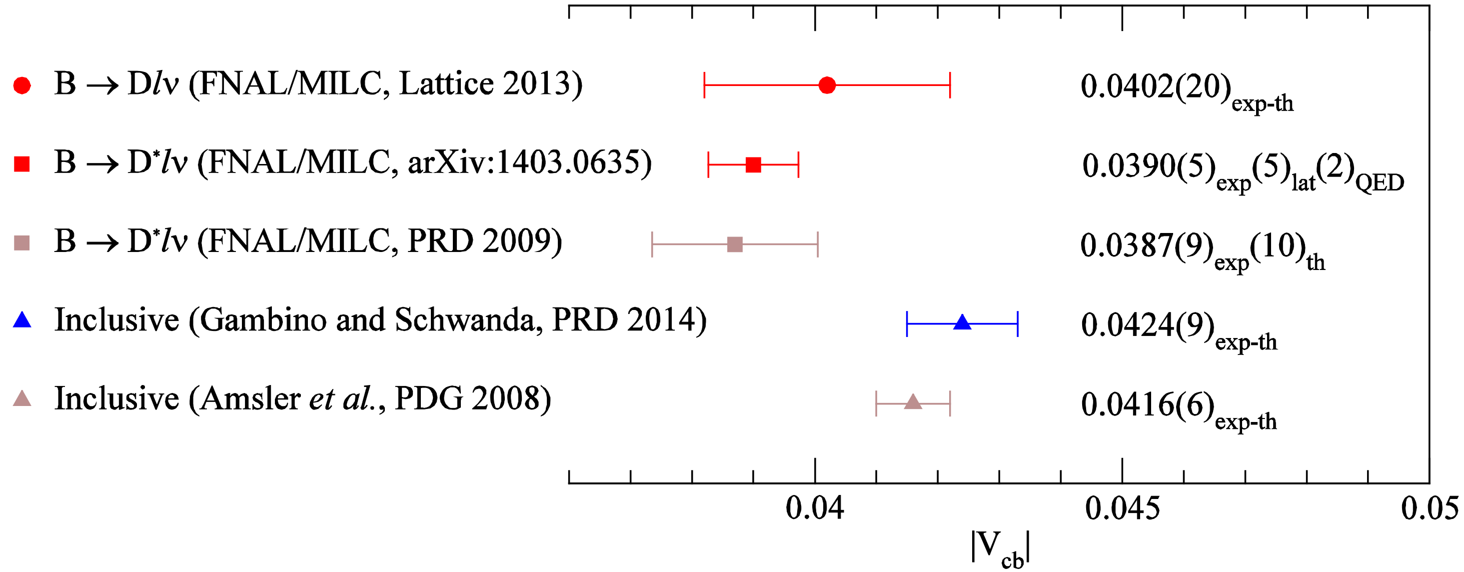
[Yong-Chull Jang *et al.*, Lattice 2013, arXiv:1311.5029; JAB *et al.*, Lattice 2014; Yong-Chull Jang *et al.*, Lattice 2014]

- $|V_{cb}|$ normalizes Unitarity Triangle ~ flavor physics
- Error in SM $\mathrm{BR}(K \to \pi\nu\bar{\nu}),\ \mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ dominated by error $|V_{cb}|$
- Error in SM ε_K dominated by error in $|V_{cb}|$
- > 3σ tension between SM and experimental $|\varepsilon_K| \sim |V_{cb}|^4$ [W. Lee *et al.*, Lattice 2014]
  - Increases with new exclusive $|V_{cb}|$ [JAB *et al.* (FNAL/MILC), PRD 2014, arXiv:1403.0635]
  - Correlated with 3.0σ difference ~ $|V_{cb}|^{excl.}$ vs. $|V_{cb}|^{incl.}$
  - Vanishes with inclusive $|V_{cb}|$



Lattice calculations

- FNAL/MILC update supersedes previous ~ first determinations of $|V_{cb}|$ from exclusive decays including vacuum polarization effects of *u*, *d*, *s* quarks
- Next generation intensity-frontier experiments, experimental errors below ~ 1%
- Lattice calculations with different discretizations of heavy quarks ~ cross checks of systematics, improved precision
- ETMC, FNAL/MILC, RBC/UKQCD, HPQCD, SWME working on $B_{(s)} \to D_{(s)}^{(*)} l\nu$ form factors for SM, BSM matrix elements [Atoui *et al*., Lattice 2013; DeTar *et al*., Lattice 2010; Kawanai *et al*., Lattice 2013; Christ *et al*., arXiv:1404.4670; Monahan *et al*., PRD 2013; Jang *et al*., Lattice 2013]



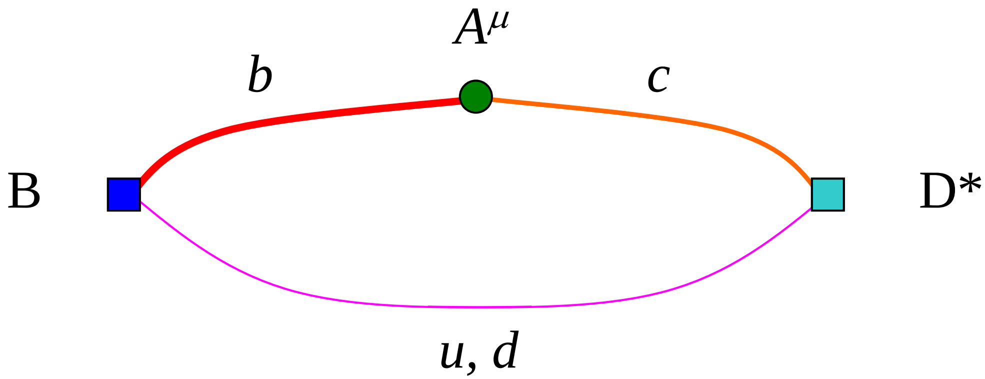
$B \to D^* l\nu$ at zero recoil

- FNAL/MILC calculations of form factor $h_{A1}(1)$

Error	PRD 2009	arXiv:1403.0635
Statistics	1.4%	0.4%
Scale (r_1) error	–	0.1%
χPT	0.9%	0.5%
$g_{D^* D\pi}$	0.9%	0.3%
Kappa tuning	0.7%	–
Discretization errors	1.5%	1.0%
Current matching	0.3%	0.4%
Tadpole tuning	0.4%	–
Isospin breaking	–	0.1%
Total	2.6%	1.4%

- "Discretization errors" are (mostly) heavy-quark discretization effects
- Chiral extrapolation errors ~ fit function and parametric uncertainties
- Parametric uncertainty from $D^*D\pi$ coupling

Strategy



- Target precision: ~ 0.7-1.0% for axial form factor at zero recoil
  - May require one-loop improvement of mass-dimension 5 operators in action
- Attack chiral extrapolation errors with physical-mass gauge ensembles
  - 2+1+1 flavor HISQ ensembles (MILC) [A. Bazavov *et al.*, PRD 2010; Lattice 2010-13]
  - Finite-volume effects for physical-mass pions [FNAL/MILC, arXiv:1403.0635]
- Reduce heavy-quark discretization effects (charm) with improved Fermilab action, currents
  - HQET power counting, $\lambda \sim a\Lambda_{QCD}, \Lambda_{QCD}/m_Q$
  - Improved action tree-level improved through $O(\lambda^3)$ in HQET [Oktay and Kronfeld, PRD 2008]
  - Axial, vector currents require improvement

Projected errors

Error	Lattice 2013	1-loop OK	tree-level OK
Statistics	0.4%	0.3%	0.3%
χPT, $g_{D^*D\pi}$	0.7%	0.3%	0.3%
Kappa tuning	0.2%	0.2%	0.2%
Discretization errors	1.0%	0.2%	0.7%
Current matching	0.5%	0.5%	0.5%
Isospin breaking	0.1%	0.1%	0.1%
Total	1.4%	0.7%	1.0%

- Discretization errors ~ power-counting estimates of heavy-quark errors
- "1-loop OK" means mass-dimension five operators in the action, corresponding operators in the current, matched at one-loop
- "tree-level OK" means tree-level matching for action, current
- Assumptions
  - 8 source times per ensemble, 1000 gauge configurations on existing HISQ ensembles, additional ensemble with lattice spacing 0.03 fm (MILC, planned for HISQ bottom)
  - Errors from statistics, kappa tuning, ChPT, $g_{DD^*\pi}$ scale with statistics
  - 50% of errors from ChPT, $g_{DD^*\pi}$ eliminated by inclusion of physical-point ensembles

Tasks

- Design B and D^* interpolating operators ← present operators suffice
- Improve current, action
  - Enumerate operators through third-order in HQET (done for OK action)
  - Match matrix elements at tree-level, one-loop (tree-level done for OK action)
- Develop code
  - Inverter (quark propagator constructor) for OK action ← optimization in progress [YCJ, WJL for SWME]
  - Application (correlator construction) code ← made available by FNAL/MILC
- Generate data
  - Kappa tuning runs ← production and preliminary analysis for tree-level OK action [YCJ for SWME, Lattice 2014]
  - Physical-mass ensembles ← HISQ ensembles publicly available [MILC]
- Calculate current renormalization factors ← independent of developing code, data production
- Analyze data
  - Correlator fits
  - Staggered chiral perturbation theory fits ← presently used formula applies for OK bottom and charm, HISQ light quarks on HISQ ensembles
  - Estimate systematics

Work in progress

- Optimization of OK inverter [Yong-Chull Jang *et al.*, Lattice 2013, arXiv:1311.5029]
  - Precalculate gauge-link combinations ~ acceleration of bi-stabilized conjugate gradient inverter
  - Wrote and tested GPU code
  - Optimizations to reduce overheads in progress
- Masses of $B_s^{(*)}$ mesons and bottomonium ~ spectrum tests of tree-level improved OK action [Yong-Chull Jang *et al.*, Lattice 2014]
  - Data for 0.12 and 0.15 fm Asqtad staggered ensembles
  - Quantify improvement (beyond clover action) in hyperfine mass splittings, inconsistency parameter [DeTar *et al.*, Lattice 2010, arXiv:1011.5189]
  - Completed preliminary tests of dispersion relations, comparisons of hyperfine splittings, reduction in inconsistency
  - Generating data for final tests, tuning [Yong-Chull Jang]
- Tree-level current improvement [JAB *et al.*, Lattice 2014]

Current improvement

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Kronfeld, PRD 2000; Harada *et al.*, PRD 2002]

- Include operators with quantum numbers of desired operator to approach continuum limit, for arbitrary quark masses

$$\mathcal{O} = Z_{\mathcal{O}}(\{m_0 a\}, g_0^2)\left[O_0 + \sum_n C_n(\{m_0 a\}, g_0^2) O_n\right]$$

- Enumerate operators ~ O(λ^3) in HQET power counting
  - O_0 ~ same dimension as continuum operator
  - O_n ~ correct deviations from continuum, suppressed or enhanced by powers of lattice spacing
- Match matrix elements to fix coefficients C_n, renormalization factor
  - Expand in coupling, external momenta
  - No expansion in quark masses, $\{m_0 a\}$

O(λ) tree-level improvement

[El-Khadra, Kronfeld, Mackenzie, PRD 1997]

- Consider continuum matrix elements of $b \longrightarrow c$ current with Dirac structure Γ, at tree-level

$$\langle c(\xi', \boldsymbol{p}')|\overline{c}\Gamma b|b(\xi, \boldsymbol{p})\rangle \rightarrow \sqrt{\frac{m_c}{E_c}}\overline{u}_c(\xi', \boldsymbol{p}')\Gamma\sqrt{\frac{m_b}{E_b}}u_b(\xi, \boldsymbol{p})$$

$$\langle 0|\overline{c}\Gamma b|b(\xi, \boldsymbol{p})\overline{c}(\xi', \boldsymbol{p}')\rangle \rightarrow \sqrt{\frac{m_c}{E_c}}\overline{v}_c(\xi', \boldsymbol{p}')\Gamma\sqrt{\frac{m_b}{E_b}}u_b(\xi, \boldsymbol{p})$$

- Standard relations for relativistic spinors, relativistic mass shell

$$u(\xi, \boldsymbol{p}) = \frac{m + E - i\boldsymbol{\gamma}\cdot\boldsymbol{p}}{\sqrt{2m(m+E)}}u(\xi, \boldsymbol{0}), \qquad E = \sqrt{m^2 + \boldsymbol{p}^2}$$

Matrix elements of lattice currents

[El-Khadra, Kronfeld, Mackenzie, PRD 1997]

- Consider matrix elements of $b \to c$ lattice current with Dirac structure Γ, at tree-level

$$\langle q_c(\xi', \boldsymbol{p}')|\overline{\psi}_c \Gamma \psi_b|q_b(\xi, \boldsymbol{p})\rangle \to \mathcal{N}_c(\boldsymbol{p}')\overline{u}_c^{\text{lat}}(\xi', \boldsymbol{p}')\Gamma \mathcal{N}_b(\boldsymbol{p}) u_b^{\text{lat}}(\xi, \boldsymbol{p})$$

$$\langle 0|\overline{\psi}_c \Gamma \psi_b|q_b(\xi, \boldsymbol{p})\overline{q}_c(\xi', \boldsymbol{p}')\rangle \to \mathcal{N}_c(\boldsymbol{p}')\overline{v}_c^{\text{lat}}(\xi', \boldsymbol{p}')\Gamma \mathcal{N}_b(\boldsymbol{p}) u_b^{\text{lat}}(\xi, \boldsymbol{p})$$

- Standard relations, relativistic mass shell altered by lattice artifacts → Lattice spinor relations, lattice mass shell ($a = 1$)

$$u^{\text{lat}}(\xi, \boldsymbol{p}) = \frac{L + \sinh E - i\boldsymbol{\gamma}\cdot\boldsymbol{K}}{\sqrt{2L(L+\sinh E)}} u(\xi, \boldsymbol{0}), \quad \cosh E = \frac{1 + \mu^2 + \boldsymbol{K}^2}{2\mu}$$

$$\mathcal{N}(\boldsymbol{p}) = \sqrt{\frac{L}{\mu \sinh E}}, \quad L = \mu - \cosh E, \quad K_i = \zeta \sin p_i$$

$$\mu = 1 + m_0 + \tfrac{1}{2} r_s \zeta \sum_i (2\sin p_i/2)^2$$

Momentum expansions

[El-Khadra, Kronfeld, Mackenzie, PRD 1997]

- Expand normalized continuum, lattice spinors for momentum small compared to $1/a$, m_q

$$\sqrt{\frac{m_q}{E}}u(\xi,\boldsymbol{p}) = \left[1 - \frac{i\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2m_q}\right]u(\xi,\boldsymbol{0}) + \mathrm{O}(\boldsymbol{p}^2)$$

$$\mathcal{N}(\boldsymbol{p})u^{\mathrm{lat}}(\xi,\boldsymbol{p}) = e^{-M_1/2}\left[1 - \frac{i\zeta\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2\sinh M_1}\right]u(\xi,\boldsymbol{0}) + \mathrm{O}(\boldsymbol{p}^2)$$

- At **p** = **0**, matrix elements differ only by normalization factor, dependent on tree-level rest mass, the lattice mass-shell energy

$$\cosh E = \frac{1+\mu^2+\boldsymbol{K}^2}{2\mu} \quad\Longrightarrow\quad e^{M_1} = 1 + m_0$$

$$Z_\Gamma \equiv e^{(M_{1c}+M_{1b})/2} \quad\Longrightarrow\quad Z_\Gamma\overline{\psi}_c\Gamma\psi_b \text{ renormalized at tree-level}$$

Improved quark field

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Kronfeld, PRD 2000; Harada *et al*., PRD 2002]

- Mismatch of matrix elements at O(**p**) remedied by improved quark field (a = 1)

$$\psi(x) \to \Psi_I(x) \equiv e^{M_1/2}[1 + d_1 \boldsymbol{\gamma} \cdot \boldsymbol{D}]\psi(x)$$
$$\overline{\psi}_c(x)\Gamma\psi_b(x) \to \overline{\Psi}_{Ic}(x)\Gamma\Psi_{Ib}(x)$$

- For tree-level matching of matrix elements of current between quark, anti-quark states, set gauge links to 1
- Note external-line factors for contractions with differentiated fields in lattice current

$$\partial_k\psi(x) \quad \Longrightarrow \quad u^{\text{lat}}(\xi, \boldsymbol{p}) \to i \sin p_k u^{\text{lat}}(\xi, \boldsymbol{p})$$

- Calculate matrix elements of improved current through O(**p**), equate continuum and lattice results to fix d_{1c}, d_{1b}

O(λ^3) tree-level improvement

- To begin, consider same current matrix elements
- Lattice spinors and mass shell modified by addition of S_6, S_7 to Fermilab action [Oktay and Kronfeld, PRD 2008]

$$K_i = \zeta \sin p_i \longrightarrow K_i = \sin p_i \left[\zeta - 2c_2 \sum_j (2\sin p_j/2)^2 - c_1 (2\sin p_i/2)^2 \right]$$

- For matching given matrix elements through O($\mathbf{p}^3$), no other modifications enter, at tree-level

- Expand normalized continuum, lattice spinors
- Examine lattice artifacts ~ deduce field improvement terms

O(λ^3) momentum expansions

- Continuum spinors through O($\mathbf{p}^3$)

$$\sqrt{\frac{m_q}{E}}u(\xi,\boldsymbol{p}) = \left[1 - \frac{i\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2m_q} - \frac{\boldsymbol{p}^2}{8m_q^2} + \frac{3i(\boldsymbol{\gamma}\cdot\boldsymbol{p})\boldsymbol{p}^2}{16m_q^3}\right] u(\xi,\mathbf{0}) + \mathrm{O}(\boldsymbol{p}^4)$$

- Lattice spinors through O($\mathbf{p}^3$)

$$\mathcal{N}(\boldsymbol{p})u^{\mathrm{lat}}(\xi,\boldsymbol{p}) = e^{-M_1/2}\left[1 - \frac{i\zeta\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2\sinh M_1} - \frac{\boldsymbol{p}^2}{8{M_X}^2} + \tfrac{1}{6}iw\gamma_k p_k^3 \right.$$
$$\left. + \frac{3i(\boldsymbol{\gamma}\cdot\boldsymbol{p})\boldsymbol{p}^2}{16{M_Y}^3}\right] u(\xi,\mathbf{0}) + \mathrm{O}(\boldsymbol{p}^4)$$

- M_X, M_Y are defined in terms of couplings m_0, ζ, r_s, c_2
- M_X, $M_Y \sim M_1$ as $a \to 0$
- w is defined in terms of m_0, ζ, c_1
- $w = r_s$ at tree-level

Improved quark field

- Inspecting momentum expansions, note independent structures of mismatches ~ one for each term at O($\mathbf{p}^2$, $\mathbf{p}^3$)
- To match matrix elements through O($\mathbf{p}^3$), consider ansatz for improved quark field (a = 1)

$$\psi(x) \to \Psi_I(x) \equiv e^{M_1/2}[1 + d_1 \boldsymbol{\gamma} \cdot \boldsymbol{D} + \tfrac{1}{2} d_2 \triangle^{(3)} + \tfrac{1}{6} d_3 \gamma_i D_i \triangle_i + \tfrac{1}{2} d_4 \{\boldsymbol{\gamma} \cdot \boldsymbol{D}, \triangle^{(3)}\}]\psi(x)$$

$$\overline{\psi}_c(x) \Gamma \psi_b(x) \to \overline{\Psi}_{Ic}(x) \Gamma \Psi_{Ib}(x)$$

- Matching O($\mathbf{p}^2$) terms yields d_2
- Matching rotation breaking terms (to zero) yields d_3
- Matching rotation preserving O($\mathbf{p}^3$) terms yields d_4

Generalized ansatz

- If Ψ_I transforms like ψ, improved current transforms correctly, and

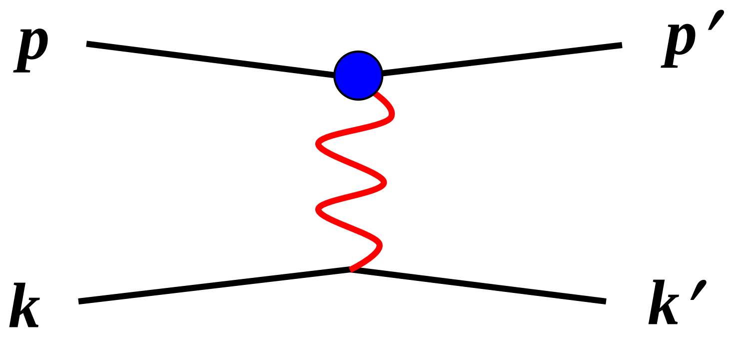
$$\overline{\psi}\Psi_I \sim \overline{\psi}\psi$$

- Enumerating all terms invariant under lattice theory symmetries through mass-dimension 6 gives

$$\begin{aligned}\psi(x) \to \Psi_I(x) \equiv e^{M_1/2}[1 &+ d_1 \boldsymbol{\gamma}\cdot\boldsymbol{D} + \tfrac{1}{2}d_2\triangle^{(3)} \\ &+ \tfrac{1}{2}i d_B \boldsymbol{\Sigma}\cdot\boldsymbol{B} + \tfrac{1}{2}d_E \boldsymbol{\alpha}\cdot\boldsymbol{E} \\ &+ \tfrac{1}{4}d_{rE}\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} + \tfrac{1}{4}d_{zE}\gamma_4(\boldsymbol{D}\cdot\boldsymbol{E} - \boldsymbol{E}\cdot\boldsymbol{D}) \\ &+ \tfrac{1}{6}d_3\gamma_i D_i \triangle_i + \tfrac{1}{2}d_4\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \triangle^{(3)}\} \\ &+ \tfrac{1}{4}d_5\{\boldsymbol{\gamma}\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\} + \tfrac{1}{4}d_{EE}\{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\} \\ &+ \tfrac{1}{4}d_{z3}\boldsymbol{\gamma}\cdot(\boldsymbol{D}\times\boldsymbol{B} + \boldsymbol{B}\times\boldsymbol{D})]\psi(x)\end{aligned}$$

Four-quark current matrix elements

- Quark scattering off current via single gluon exchange



- Diagram vanishes in QCD
- Non-trivial with improved current
- Calculate on-shell matrix element, expand in momenta of external quarks, match to extract d_i
- Improved field suffices for $O(\lambda^3)$-improved current?

Summary and outlook

- Bright future for lattice QCD and experimental efforts, understanding of quark flavor, CP violation, search for new physics
- $|V_{cb}|$ crucial in this era; general methods for improvement ~ EFTs
  - Tension in ε_K with exclusive $|V_{cb}|$ suggests significant tension in UT analysis with exclusive $|V_{cb}|$ (and exclusive $|V_{ub}|$)
  - Parametric uncertainty in $K \to \pi \nu\nu$-bar, $B_s \to \mu^+\mu^-$
  - Improved Fermilab action with tree-level matching through third order in HQET [Oktay and Kronfeld, PRD 2008, arXiv:0803.0523]
  - Improved Fermilab currents . . . [JAB *et al.*, Lattice 2014]
- Increasingly precise determinations of form factors for $B \to \pi l\nu$, $B_s \to K l\nu$, $B \to D^{(*)} l\nu$, $D \to \pi l\nu$, $D \to K l\nu$, $B \to \pi l^+ l^-$, $B \to K l^+ l^-$ yielding new information about $|V_{ub}|$, $|V_{cb}|$, $|V_{cs}|$, $|V_{cd}|$, tests of lattice methods, and SM
- Lattice calculations of kaon mixing BSM *B*-parameters yet to converge
- Lattice calculations of golden semileptonic decay form factors mature, entering second generation ~ sub-percent precision